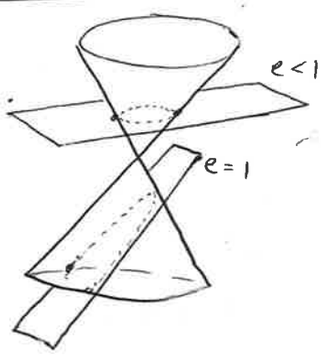
§10.1

MAIN CONCEPTS

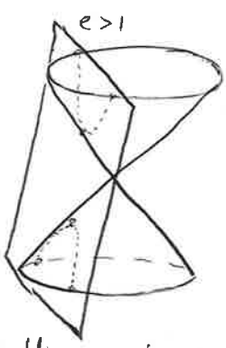
1. Conic Sections
2. $e=1$ standard form

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1. For the next few sections, we'll study conic sections, the intersection of a (double-) cone with a plane. This divides up into 3 cases.



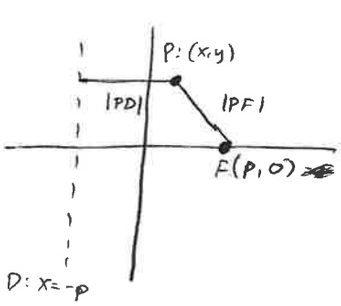
When the plane is relatively "horizontal", it only intersects one of the two cone halves, so the intersection is some kind of stretched circular shape. As the plane tilts more, there's a single angle when it becomes parallel to the edge of the cone. Finally, once the plane is relatively "vertical", it intersects both halves of the double-cone.



We'll encode this with a number e, called the eccentricity of the conic section. $e=1$ is the parallel case, with $e<1$ being more horizontal and $e>1$ being more vertical. By doing some geometry, we can identify that each conic section has an important point, the focus F, and an important line, the directrix D. All of these fit together into the equation:

CONIC SECTIONS: A conic section consists of all points P satisfying $|PF| = e|PD|$, where e = eccentricity, F = focus, D = directrix.

2. In this section, we'll handle $e=1$. To simplify things, we'll pick the focus to be $F=(p,0)$ and the directrix to be the line $x=-p$. Then we want $|PF| = e\cdot|PD|$, i.e. $|PF| = |PD|$ since $e=1$. Giving P the coordinates (x,y), we get



$|PF| = \sqrt{(x-p)^2+(y-0)^2} = \sqrt{(x-p)^2+y^2}$ and

$|PD| = |(x)-(-p)| = |x+p|$. So $\sqrt{(x-p)^2+y^2} = |x+p|$. Squaring gives

$$(x-p)^2+y^2 = (x+p)^2 \longrightarrow x^2-2px+p^2+y^2 = x^2+2px+p^2 \longrightarrow -2px+y^2 = 2px$$

$$\longrightarrow \boxed{y^2 = 4px}$$

We recognize this: it's a parabola! We've deduced the standard form for a parabola.

PARABOLA STANDARD FORM: The parabola $y^2 = 4px$ has focus $(p,0)$ and directrix $x=-p$.

EXAMPLES: Locate the focus and directrix of each parabola below.

① $y^2 = 8x$.

This is in standard form with $p=2$, so the focus is at $(2,0)$ and directrix is $x=-2$.

(2) $2y^2 + 10x = 0$

We need to manipulate this into standard form: $2y^2 = -10x \longrightarrow y^2 = -5x = 4px$

So $4p = -5$ and $p = -\frac{5}{4}$. So the focus is at $(-\frac{5}{4}, 0)$ and the directrix is $x = \frac{5}{4}$.

(3) $y = x^2$

This is the parabola we're used to, but we can't put this in standard form with just algebra. Instead, we use some geometry: switching x & y will flip things back in place. So we solve the modified case first.

[Modify (swap $x \longleftrightarrow y$): $x = y^2 \longrightarrow y^2 = 4(\frac{1}{4})x \rightarrow$ focus $(\frac{1}{4}, 0)$ directrix $x = -\frac{1}{4}$

Unmodify (swap $x \longleftrightarrow y$ back): focus $(0, \frac{1}{4})$ and directrix $y = -\frac{1}{4}$.

Visually, the examples above are:

