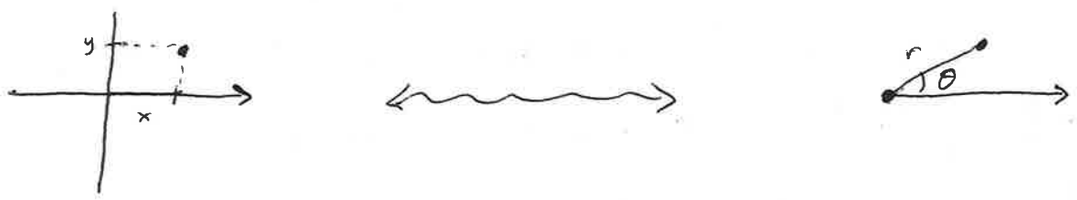
§10.5

MAIN CONCEPT

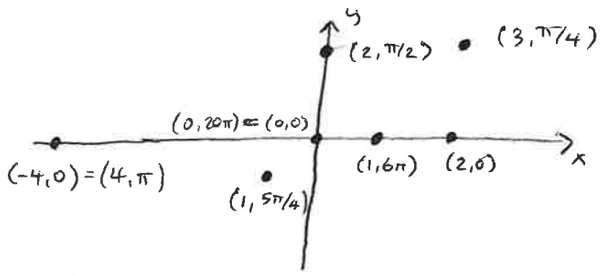
1 Polar coordinates

1 In 2D space, we usually identify points by specifying 2 numbers; a horizontal distance (x) and a vertical distance (y). There are other choices, however. We alluded to one earlier with rotated "u-v" axes. Another common choice is given in <u>polar coordinates</u>, given by specifying a direction (θ) and a distance (r).



EXAMPLES: Plot the following points (r, θ):

① $(2, 0)$
② $(2, \pi/2)$
③ $(3, \pi/4)$
④ $(1, 5\pi/4)$
⑤ $(1, 6\pi)$
⑥ $(0, 0)$
⑦ $(0, 20\pi)$
⑧ $(-4, 0)$
⑨ $(4, \pi)$



(Notice that the angle isn't unique since we can $\pm 2\pi$, r isn't unique because it can be $\pm$, and $r=0$ doesn't care about θ)

EXAMPLES: Graph the polar equations

① $r = 3$

This is saying: plot all points (r, θ) with $r = 3$, so plotting a few gives a clear guess: this is a circle.



② $r = 2\sin\theta$

Let's make a table of points:

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$
r	0	$\sqrt{2}$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$

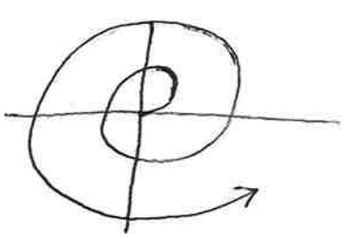


This actually only gives 4 points (plot them), so we maybe can't guess as easily.

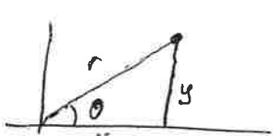
③ $r = \theta$, $\theta \geq 0$

As the angle increases, so does the distance r.

So this graph spirals outwards.



To gain some confidence about ②, it would help to convert back to Euclidean (x, y) coordinates. To do so, we refer to the following diagram. From it, we get:



EUCLIDEAN-POLAR CONVERSIONS: $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2+y^2$, $\tan\theta = \frac{y}{x}$.

EXAMPLES: Convert points to the opposite coordinate system:

① $(r,\theta) = (2, \pi/2)$

This is just plugging in: $(x,y) = (r\cos\theta, r\sin\theta) = (2\cos(\frac{\pi}{2}), 2\sin(\frac{\pi}{2})) = \boxed{(0,2)}$.

② $(x,y) = (1,1)$.

In this direction, there are choices, since neither r nor θ is uniquely determined. We have $r^2 = x^2+y^2 = 1^2+1^2 = 2$ so $r = \pm\sqrt{2}$. We'll choose $r = \sqrt{2}$. Then $\tan(\theta) = \frac{y}{x} = \frac{1}{1} = 1$. So $\theta = \ldots, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \ldots$ Since $(1,1)$ is in the first quadrant, we need $\theta = \ldots, \frac{\pi}{4}, \frac{9\pi}{4}, \ldots$ so we'll choose $\boxed{(r,\theta) = (\sqrt{2}, \frac{\pi}{4})}$. To demonstrate, the following are also acceptable: $(r,\theta) = (\sqrt{2}, \frac{9\pi}{4}) = (-\sqrt{2}, -\frac{3\pi}{4}) = (-\sqrt{2}, \frac{5\pi}{4})$.

EXAMPLES: Convert equations to the opposite coordinates.

① $r = 3$.

Squaring gives $r^2 = 9 \longrightarrow x^2+y^2 = 9 \longrightarrow \frac{x^2}{9} + \frac{y^2}{9} = 1$. So this is an ellipse, but both diameters are equal, so it's a circle as claimed.

② $r = 2\sin\theta$.

Multiply by r: $r^2 = 2r\sin\theta \longrightarrow x^2+y^2 = 2y \longrightarrow x^2+y^2-2y = 0 \longrightarrow$

$\longrightarrow x^2+(y^2-2y+1) = 1 \longrightarrow x^2+(y-1)^2 = 1$.

This is again a circle, centered at $(0,1)$, so we've answered the previous page.

③ $y = 1-x$.

Plug it in directly: $r\sin\theta = 1-r\cos\theta$. Typically, we solve for r to get:

$\longrightarrow r\sin\theta + r\cos\theta = 1 \longrightarrow r(\sin\theta+\cos\theta) = 1 \longrightarrow r = \frac{1}{\sin\theta+\cos\theta}$.

④ $y = 4x^2$

Ditto: $r\sin\theta = 4(r\cos\theta)^2 \longrightarrow r\sin(\theta) = 4r^2\cos^2(\theta) \longrightarrow \sin\theta = 4r\cos\theta \longrightarrow$

$\longrightarrow r = \frac{\sin\theta}{4\cos\theta} = \frac{1}{4}\tan\theta$.