§11.2

Main Concepts

1. Soapbox — Algebra ↔ Geometry
2. Definitions & operations with vectors
3. Unit vectors & standard basis.

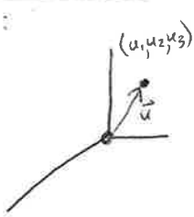
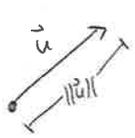
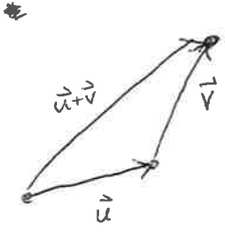
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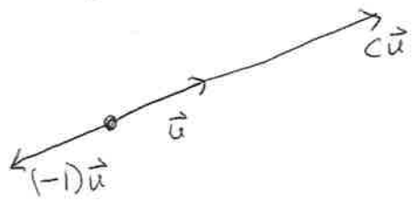
1 I often don't include my "speeches" in the lecture notes, but this one is important:

The rest of this class concerns the interplay between geometry and algebra. Vectors are one incarnation of this, where some problems are easier algebraically and some are easier geometrically. Our goal, then, as students of mathematics, is to internalize both approaches — any time we introduce a new idea, we should ask: "Where's the geometry? Can I draw a picture?" and "Where's the algebra? What is known and what do I want to solve for?"

(/soapbox)

2

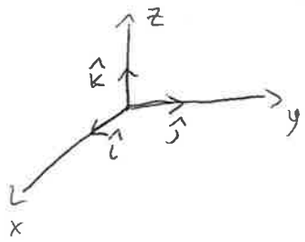
Term/idea	Geometry	Algebra
Vector	An object with direction and magnitude (but without position/location). Usually depicted by an arrow: 	An x,y,z-coordinate triple: $\vec{u} = \langle u_1, u_2, u_3 \rangle$. This is the vector pointing from $(0,0,0)$ to (u_1,u_2,u_3) 
Magnitude	The length of the arrow. Denoted by $\|\vec{u}\|$. 	From the geometric definition and the distance formula, we get a formula: $\|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$.
Sum	The vector $\vec{u}+\vec{v}$ is the total you get by "traveling along" $\vec{u}$ and then $\vec{v}$ 	Algebraically, we add each component: $\langle u_1, u_2, u_3 \rangle + \langle v_1, v_2, v_3 \rangle$ $= \langle u_1+v_1, u_2+v_2, u_3+v_3 \rangle$

Scaling	The vector you get by scaling the magnitude but not the direction (except with negative numbers): 	Obtained by multiplying coordinates: $c\langle u_1, u_2, u_3\rangle$ $= \langle cu_1, cu_2, cu_3\rangle$.

3 A unit vector is any vector with magnitude 1. Often we use a hat ($\hat{}$) instead of an arrow ($\rightarrow$) when writing unit vectors. There are 3 important ones:

$$\hat{\imath} = \langle 1,0,0\rangle \qquad \hat{\jmath} = \langle 0,1,0\rangle \qquad \hat{k} = \langle 0,0,1\rangle.$$

That is, they are the vectors pointing 1 unit in the x, y, and z directions, respectively:



Sometimes these can be useful algebraically. For example:

$$\langle u_1, u_2, u_3\rangle = \langle u_1, 0, 0\rangle + \langle 0, u_2, 0\rangle + \langle 0, 0, u_3\rangle$$
$$= u_1\langle 1,0,0\rangle + u_2\langle 0,1,0\rangle + u_3\langle 0,0,1\rangle$$
$$= u_1\hat{\imath} + u_2\hat{\jmath} + u_3\hat{k}$$

So, if preferred, we don't need to write a vector with angle brackets $\langle , \rangle$ and can use $\hat{\imath}, \hat{\jmath}, \hat{k}$ instead.

Finally, unit vectors encode directions, so we often want a unit vector pointing in the same direction as a given vector $\vec{u}$. For this, divide (scale down) by $\|\vec{u}\|$:

Unit Vector Formula $\quad \hat{u} = \dfrac{\vec{u}}{\|\vec{u}\|}$.

Examples: Find $\hat{u}$, given $\vec{u}$:

① $\vec{u} = \langle 2,2,1\rangle$

$\|\vec{u}\| = \sqrt{2^2+2^2+1^2} = \sqrt{9} = 3$

$\hat{u} = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\rangle$

② $\vec{u} = \langle -3,4,0\rangle$

$\|\vec{u}\| = \sqrt{(-3)^2+4^2+0^2} = \sqrt{25} = 5$

$\hat{u} = \langle -\frac{3}{5}, \frac{4}{5}, 0\rangle$

③ $\vec{u} = \langle 1,1,1\rangle$

$\|\vec{u}\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\hat{u} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\rangle$