§11.8

Main Concepts

1. Cross Sections
2. Cylinders
3. Quadric Surfaces

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1 In 3D, the graph obtained from one equation is typically 2-dimensional. In other words, it defines a surface. We've seen two such examples: spheres $[x^2+y^2+z^2=1]$ and planes $[Ax+By+Cz=D]$.

To understand other examples, we would like to be able to graph them. One useful strategy is to take cross sections of the surface — the intersection of the surface with a plane. Often, planes parallel to the xy-, yz-, and xz-planes are useful.

Examples: Compute some cross sections and sketch each surface

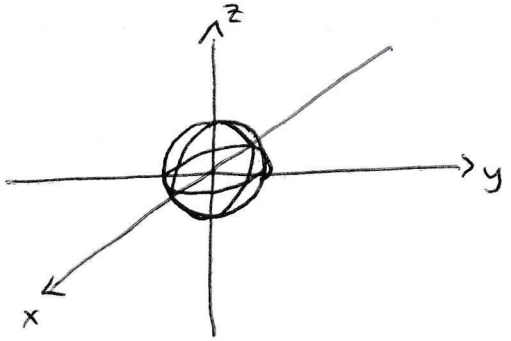
① $x^2+y^2+z^2=1$.

We already know this is a sphere, but let's try to rediscover this. Cross-sections:

$x=0 \longrightarrow y^2+z^2=1$ circle of radius 1 in yz-plane

$y=0 \longrightarrow x^2+z^2=1$ circle of radius 1 in xz-plane

$z=0 \longrightarrow x^2+y^2=1$ Circle of radius 1 in xy-plane



Indeed, this resembles a sphere!

② $z = x^2 + y^2$

This we don't immediately recognize. Let's take cross-sections:

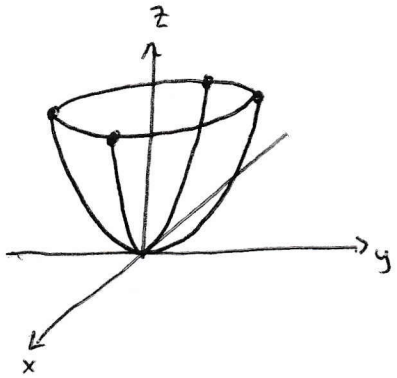
$x=0 \longrightarrow z = y^2$ Parabola in yz

$y=0 \longrightarrow z = x^2$ Parabola in xz

$z=0 \longrightarrow x^2+y^2=0$ Just the origin since $x^2 \geq 0$, $y^2 \geq 0$.

This last cross section wasn't as useful, since it was just one point. So, let's try another "z"-cross section.

$z=1 \longrightarrow x^2+y^2=1$ Circle of radius 1



③ $y = x^2$

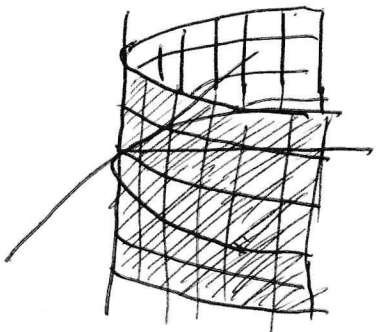
This looks like the equation of a parabola! But we are still in 3D, so what's the role of z? Well, any "z"-cross section is the same:

$z=0 \longrightarrow y = x^2$

$z=1 \longrightarrow y = x^2$

$z=k \longrightarrow y = x^2$

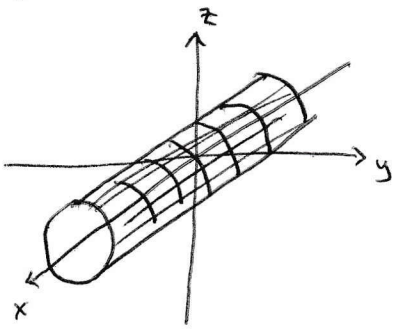
So, the graph is this same parabola "swept" vertically.


(Shading for emphasis).

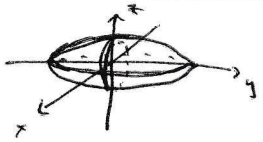
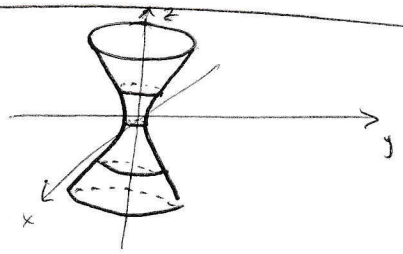
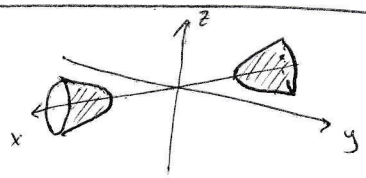
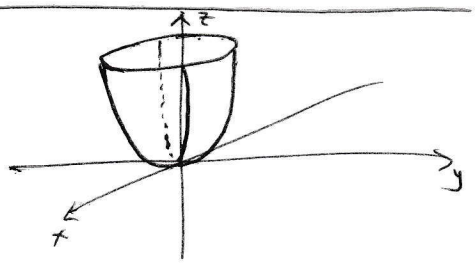
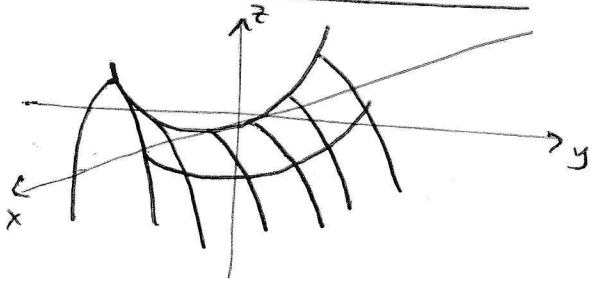
2 This last example is a general phenomenon. If we have an equation involving two variables, then the missing variable can take any value. So, the graph of the surface is given by the 2D graph of the curve "swept" along the axis of the missing variable. We call any such surface a "cylinder."

Example: Graph the following surface: $y^2+z^2=4$.

Since x is missing, this is a cylinder in the x-direction. $y^2+z^2=4$ is a circle of radius 2 so we get a "usual" cylinder (i.e. one with a circular base).



3 Finally, we finish with common surfaces and their names. EXERCISE: Reproduce the sketches below from just the equations.

Equation	Name of surface	Sketch
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Ellipsoid	
$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	Hyperboloid I	
$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	Hyperboloid II	
$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	Paraboloid	
$z=\frac{x^2}{a^2}-\frac{y^2}{b^2}$	Saddle	
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$	Cone	