§12.1

Main Concepts

1. Functions of multiple variables
2. Visualizing functions of 2 variables
3. Visualizing functions of 3 variables

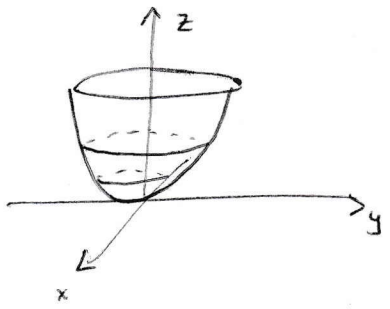
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1 Until now, we've mostly dealt with functions $f(x)$ with one input and one output, or, more recently, $\vec{r}(t)$ which has one input and 2 or 3 outputs. We can also have functions of multiple variables, such as $T(x,y,z)$, the temperature at a location (x,y,z) or $A(b,h) = \frac{1}{2}bh$, the area of a triangle with base b and height h.

2 Given a function $f(x,y)$, we can understand it better by graphing it. Just like the 1-dimensional case, graphing it requires introducing a new variable [in 1-dimension: $y=f(x)$ gives a 2D graph]. So, we write $z=f(x,y)$ and graph the resulting surface.

Example: Graph $f(x,y) = x^2 + \frac{y^2}{4}$.

We write $z=f(x,y) \longrightarrow z = x^2 + \frac{y^2}{4}$ which is a paraboloid.



Another way to understand a function $f(x,y)$ is to study the set of points where $f(x,y) = \text{constant}$. These are called <u>level sets</u>, and in the 2D case we call them <u>level curves</u>.

Examples: Draw some level curves for each function f.

① $f(x,y) = x^2+y^2$

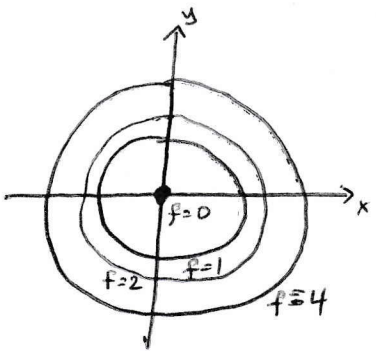
We write $f(x,y)=k$ for different k:

$k=0$: $x^2+y^2=0$ → Only the point $(0,0)$

$k=1$: $x^2+y^2=1$ → Circle of radius 1

$k=2$: $x^2+y^2=2$ → Circle of radius $\sqrt{2}$

$k=-1$: $x^2+y^2=-1$ → No solution.



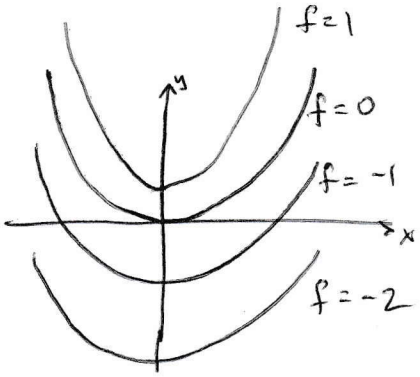
② $f(x,y) = y-x^2$

Same idea:

$k=0$: $y-x^2=0 \rightarrow y=x^2$ → Parabola

$k=-1$: $y-x^2=-1 \rightarrow y=x^2-1$ → Parabola shifted down

$k=1$: $y-x^2=1 \rightarrow y=x^2+1$ → Parabola shifted up.



③ For a function $f(x,y,z)$ of 3 variables, we can't use the first strategy since $w=f(x,y,z)$ is 4-dimensional (so hard to graph). But we can still look at level sets, called <u>level surfaces</u> in the 3D case.

Example: Graph level surfaces of $f(x,y,z) = x^2+z^2-y$.

Again, we set $f(x,y,z)=k$:

$k=-1$: $x^2+z^2-y=-1 \rightarrow y=x^2+z^2+1$ → Paraboloid

$k=0$: $x^2+z^2-y=0 \rightarrow y=x^2+z^2$ → Paraboloid

General k: $x^2+z^2-y=k \rightarrow y=x^2+z^2-k$ → Paraboloid

