§2.1

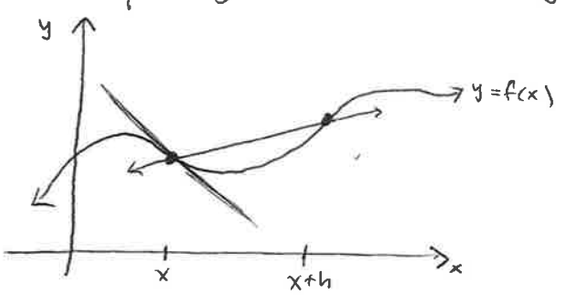
Main Concept

1 Rate of change; slope

1 If $f(x)$ is a function, we're often interested not only in the values of $f(x)$, but also in the rate of change of $f(x)$ as x varies. For example, if $f(x)$ is the location of an object at time x, we can ask how far the object moves in 1 second. Or, if $f(x)$ is the number of bacteria in a petri dish after x hours, we can ask how quickly the colony is growing or shrinking per minute. Or, if $f(x)$ is the cost of a car, we can ask if the cost is increasing, decreasing, or staying steady.

In all of these questions, we're comparing the current state ($f(x)$) at time x to a future state ($f(x+h)$) after "h" units of time. Of course, the difference depends on h — you can travel further in a minute than in a second, — but this suggests studying $\dfrac{f(x+h)-f(x)}{h}$, the $\dfrac{\text{"change in } f\text{"}}{\text{"change in } x\text{"}}$ or rate of change.

We've actually seen this quantity before. If we graph $y=f(x)$, then the two



points $(x, f(x))$ and $(x+h, f(x+h))$ lie on the graph, and the slope of the line between them is $\dfrac{y_2-y_1}{x_2-x_1} = \dfrac{f(x+h)-f(x)}{(x+h)-(x)} = \dfrac{f(x+h)-f(x)}{h}$.

Accordingly, this quantity is the average rate of change or average slope. If we want to understand just the point x, we should pick a smaller h. For example, to find your speed in a car, you could ask how many miles you've driven in the last hour. But if you were speeding up or slowing down, this could fluctuate. So, to get your current speed, it would be better to ask about the last minute or last second or even the last millisecond. In other words, the instantaneous velocity or instantaneous slope or instantaneous rate of change is $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.