§2.6

MAIN CONCEPTS

1. Higher derivatives
2. Interpretation

---

1. If $f(x)$ is a function, we've established some rules to compute $f'(x)$. The result is also a function, so we can take its derivative, and so on. The "derivative of the derivative" is called the 2nd derivative and comes with new notation:

NAME	"Prime notation"	Leibniz notation	"Action" notation
Derivative	$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}(\quad)$
2nd derivative	$f''(x)$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}(\quad)$
3rd derivative	$f'''(x)$ OR $f^{(3)}(x)$	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}(\quad)$
⋮	⋮	⋮	
100th derivative	$f^{(100)}(x)$	$\frac{d^{100}y}{dx^{100}}$	$\frac{d^{100}}{dx^{100}}(\quad)$

EXAMPLES: Compute the first 3 derivatives in each example.

1. $f(x) = x^4 - 3x^3 + 7x^2 + 10x - 1000$ — There's no new skill; just repetition!

$f'(x) = 4x^3 - 9x^2 + 14x + 10$ [Power rule]

$f''(x) = 12x^2 - 18x + 14$

$f'''(x) = 24x - 18$

2. $y = \sin(2x)$

$\frac{dy}{dx} = 2\cos(2x)$ [chain rule]

$\frac{d^2y}{dx^2} = -2\sin(2x)\cdot 2 = -4\sin(2x)$

$\frac{d^3y}{dx^3} = -4(\cos(2x))\cdot(2) = -8\cos(2x)$.

3. $f(x) = x\sin(x)$

$f'(x) = \sin(x) + x\cos(x)$ [Product rule]

$f''(x) = \cos(x) + [\cos(x) + x(-\sin(x))]$

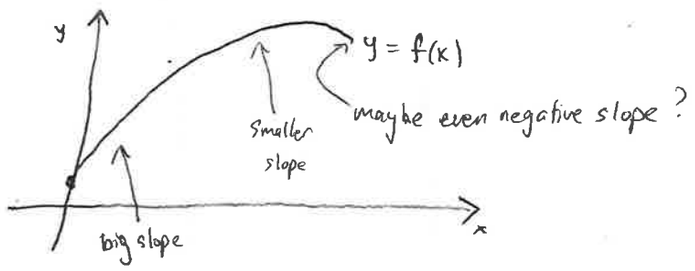
$= 2\cos(x) - x\sin(x)$

$f'''(x) = -2\sin(x) - [\sin(x) + x\cos(x)]$

$= -3\sin(x) - x\cos(x)$

2 So, we can compute more derivatives, but why should we? The point is to understand functions even better. If $f'(x) > 0$, then we know that $f(x)$ is increasing. But if $f''(x) < 0$, this means $f'(x)$ is decreasing, so even though it's positive now, it might be negative later.

For example, if $f(t)$ measures the cost of gas on day x, and $f'(x) > 0$, you might decide to stock up now — prices are going up so today's the cheapest. But if $f''(x) < 0$, then $f'(x)$ is decreasing, so even though prices are increasing, they're slowing down and might even start decreasing soon. Graphically, it might look like:



Another common application comes from physics. If $f(t)$ is your location at time t, then we've already seen that $f'(t)$ is your velocity. This means that $f''(t)$ is the rate at which your speed changes — acceleration. If $f''(t) > 0$, it means you're speeding up, while $f'(t) > 0$ just means you're moving forward.