§7.1

MAIN CONCEPTS

1. Substitution
2. Memorization

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1. This section is largely review. Mainly, it reminds us how to do substitution (or "u-sub") to solve integrals.

EXAMPLES: Solve each integral using a substitution.

① $\int x^2 \cos(x^3+2)\,dx$

We see an "inside function" and its derivative, an excellent sign of u-sub. So choose $u = x^3+2$ and differentiate to get $du = 3x^2\,dx$. So, we get $x^2\,dx = \frac{1}{3}du$ and

$$\int x^2\cos(x^3+2)\,dx = \frac{1}{3}\int \cos(u)\,du = \frac{1}{3}\sin(u) + C = \frac{1}{3}\sin(x^3+2) + C.$$

② $\int_1^{10} \frac{\ln(x)}{x}\,dx$

Same idea: choose $u = \ln(x)$ so $du = \frac{1}{x}dx$. Now we have a definite integral, so we should also convert the bounds: $x=10 \to u = \ln(10)$, $x=1 \to u = \ln(1) = 0$. So:

$$\int_1^{10} \frac{\ln(x)}{x}\,dx = \int_0^{\ln(10)} u\,du = \frac{1}{2}u^2\Big|_0^{\ln(10)} = \frac{1}{2}(\ln(10))^2 - \frac{1}{2}(0)^2 = \frac{1}{2}(\ln(10))^2.$$

2. The other note in this section is that if we're going to integrate, we need some integrals memorized, or at least on-hand. Here's a list (not necessarily comprehensive) of useful integrals:

- $\int x^n\,dx = \begin{cases} \frac{1}{n+1}x^{n+1}+C & \text{if } n \neq -1 \\ \ln|x|+C & \text{if } n=-1 \end{cases}$
- $\int \sin(x)\,dx = -\cos(x)+C$
- $\int \tan(x)\,dx = -\ln|\cos(x)|+C$
- $\int \csc(x)\,dx = \ln|\csc(x)-\cot(x)|+C$
- $\int \sec^2(x)\,dx = \tan(x)+C$
- $\int \csc^2(x)\,dx = -\cot(x)+C$
- $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x)+C$
- $\int \sinh(x)\,dx = \cosh(x)+C$
- $\int e^x\,dx = e^x+C$
- $\int \cos(x) = \sin(x)+C$
- $\int \sec(x)\,dx = \ln|\sec(x)+\tan(x)|+C$
- $\int \cot(x)\,dx = \ln|\sin(x)|+C$.
- $\int \sec(x)\tan(x)\,dx = \sec(x)+C$
- $\int \csc(x)\cot(x)\,dx = -\csc(x)+C$
- $\int \frac{1}{1+x^2}\,dx = \arctan(x)+C$
- $\int \cosh(x)\,dx = \sinh(x)+C$.