Exercise (4.1.1).

Proof. We are given a finite group G, a subgroup H, a representation (U,τ) of G, and a representation (V,π) of H. First, I claim that the map $\mathrm{Hom}_G(U,V^G)\to\mathrm{Hom}_H(U_H,V)$ given by:

$$\Phi\mapsto(u\mapsto\Phi(u)(e)$$

is a $\mathbb{C}$-linear transformation, where e denotes the identity of G. Linearity is clear, so we only need to show that the image of Φ is an H-equivariant map. For $h\in H$ and $u\in U$, we have:

$$\Phi(\tau(h)u)(e)=(\pi^G(h)\Phi(u))(e)=\Phi(u)(eh)=\Phi(u)(he)=\pi(h)\Phi(u)(e)$$

as desired.

Conversely, we claim that the map $\mathrm{Hom}_H(U_H,V)\to\mathrm{Hom}_G(U,V^G)$ given by:

$$\Psi\mapsto(u\mapsto(g\mapsto\Psi(\tau(g)u)))$$

is also $\mathbb{C}$-linear. Linearity is again clear, so we wish to show that the image of Ψ is G-equivariant and that this image evaluated at u is an element of V^G for each u. For both, it will be convenient for a fixed Ψ to define $f_u:G\to V$ by:

$$f_u(g)=\Psi(\tau(g)u)$$

for each $u\in U$. Then the map above can be defined by $\Psi\mapsto(u\mapsto f_u)$. To see that f_u is in V^G, note that for any $h\in H$ and $g\in G$,

$$f_u(hg)=\Psi(\tau(hg)u)=\Psi(\tau(h)(\tau(g)u))=\pi(h)\Psi(\tau(g)u)=\pi(h)f_u(g)$$

as desired. Finally, we show that the overall map is G-equivariant; for $g,g'\in G$ and $u\in U$, we have:

$$f_{\tau(g')u}(g)=\Psi(\tau(g)(\tau(g')u))=\Psi(\tau(gg')u)=f_u(gg')=(\pi^G(g')f_u)(g)$$

So that $f_{\tau(g')u}=\pi^G(g')f_u$ as desired.

Finally, to complete the proof, we show these two morphisms are inverses. For $\Phi\in\mathrm{Hom}_G(U,V^G)$, let Ψ be the image over the first morphism and Φ' be the image of Ψ across the second. Then, for $u\in U$ and $g\in G$,

$$\Phi'(u)(g)=\Psi(\tau(g)u)=\Phi(\tau(g)u)(e)=(\pi^G(g)\Phi(u))(e)=\Phi(u)(eg)=\Phi(u)(g)$$

so that $\Phi'=\Phi$. Conversely, for $\Psi\in\mathrm{Hom}_H(U_H,V)$, let Φ be the image over the second and Ψ' be the image of Φ over the first. Then, for $u\in U$,

$$\Psi'(u)=\Phi(u)(e)=\Psi(\tau(e)u)=\Psi(u)$$

so $\Psi'=\Psi$. Hence both compositions are the identity, and we have $\mathrm{Hom}_G(U,V^G)\cong\mathrm{Hom}_H(U_H,V)$ as $\mathbb{C}$-vector spaces.

Now let (π_1,V_1) and (π_2,V_2) be representations of the same finite group G with characters χ_1,χ_2. As per the hint, we know that both $\langle\chi_1,\chi_2\rangle_G$ and $\dim\mathrm{Hom}_G(\pi_1,\pi_2)$ are linear in each coordinate. Namely, if π_1 is the direct sum of sub-representations, then χ_1 is the corresponding sum of characters, the inner product expands as a sum, the vector space of homs splits as a direct sum since each map can be defined on each summand, and the dimension adds. Similarly, using the projections, both terms are linear in π_2. So, showing these quantities are equal reduces to the case when π_1,π_2 are irreducible.

Let $S\in\mathrm{Hom}_G(\pi_1,\pi_2)$. Then note that $\ker S$ is a π_1-equivariant subspace of V_1, and $\mathrm{im}\,S$ is a π_2-equivariant subspace. Indeed, for $v\in\ker S$, $w=Sz\in\mathrm{im}\,S$, and $g\in G$ arbitrary:

$$S\pi_1(g)v=\pi_2(g)Sv=\pi_2(g)0=0$$
$$\pi_2(g)w=\pi_2(g)Sz=S\pi_1(g)z$$

so that $\pi_g(v)\in\ker S$ and $\pi_2(g)w\in\mathrm{im}\,S$. By irreducibility, this shows that the kernel and image are trivial. If either $\ker S=V_1$ or $\mathrm{im}\,S=0$, then $S=0$ is the zero map. Otherwise $\ker S=0$ and $\mathrm{im}\,S=V_2$ and S is an isomorphism. So each nonzero element of $\mathrm{Hom}_G(\pi_1,\pi_2)$ is an isomorphism of representations.

We now have two cases. If $\pi_1\ncong\pi_2$, then the above shows that $\mathrm{Hom}_G(\pi_1,\pi_2)=0$, which has dimension zero. But in this case we also know $\langle\chi_1,\chi_2\rangle=0$, so the two quantities are equal as claimed. Otherwise $\pi_1\cong\pi_2$, and we can pick an explicit isomorphim T. We know $\langle\chi_1,\chi_2\rangle=1$ in this case, so it suffices to show that $\mathrm{Hom}_G(\pi_1,\pi_2)=\mathbb{C}T$. To this end, let

$S \in \mathrm{Hom}_G(\pi_1, \pi_2)$. Either $S = 0 = 0T \in \mathbb{C}T$, or S is an isomorphism. Then $T^{-1}S$ is a π_1-equivariant automorphism of V_1. Let λ, v be an eigenvalue-eigenvector pair of $T^{-1}S$. Then

$$\mathrm{span}\{\pi_1(g)v \mid g \in G\}$$

is a nonzero π_1-invariant subspace of V_1 (since it contains v), so it must be all of V_1 by irreducibility. So, for $w \in V_1$, we can write:

$$w = \sum_i c_i \pi_1(g_i) v$$

for some finite collection of $c_i \in \mathbb{C}$ and $g_i \in G$. Then,

$$T^{-1}Sw = T^{-1}S \sum_i c_i \pi_1(g_i) v = \sum_i c_i (T^{-1}S\pi_1(g_i))v = \sum_i c_i(\pi_1(g)T^{-1}S)v = \sum_i c_i \pi_1(g_i)\lambda v = \lambda \sum_i c_i \pi_1(g_i) v = \lambda w$$

So that $T^{-1}S = \lambda I$, i.e. $S = \lambda T \in \mathbb{C}T$, completing the proof.

Finally, the last statement combines these two trivially:

$$\left\langle \sigma, \chi^G \right\rangle = \dim \mathrm{Hom}_G(\tau, \pi^G) = \dim \mathrm{Hom}_H(\tau_H, \pi) = \langle \sigma_H, \chi \rangle$$

as claimed. □

Exercise (4.1.2).

Proof. We would like to show $\Delta(x_i)$ is S_i-equivariant for each i. For this, suppose $g \in S_i$. Then,

$$\Delta(x_i) \circ \pi_{1,i}(g) = \pi_2(e) \circ \Delta(x_i) \circ \pi_1(x_i^{-1} g x_i) = \Delta(e x_i x_i^{-1} g x_i) = \Delta(g x_i e) = \pi_2(g) \circ \Delta(x_i) \circ \pi_1(e) = \pi_{2,i}(g) \circ \Delta(x_i)$$

as desired.

This now suggests that we can define $\Phi : \mathrm{Hom}_G(V_1^G, V_2^G) \to \bigoplus_{i=1}^r \mathrm{Hom}_{S_i}(\pi_{1,i}, \pi_{2,i})$ by:

$$\Phi(\Delta) = (\Delta(x_1), \ldots, \Delta(x_r))$$

where we identify $\mathrm{Hom}_G(V_1^G, V_2^G)$ with the vector space of functions Δ satisfying equation (1.4). The above computation shows that we have correctly stated the codomain. It is also clear that Φ is linear.

So, we now wish to show it is bijective. For injectivity, suppose $\Delta \in \ker \Phi$, whence $\Delta(x_i) = 0$ for all i. Now, let $g \in G$, and note that $g = h_2 x_i h_1$ for some $h_2 \in H_2$ and $h_1 \in H_1$ since we have chosen the x_i to represent the double cosets. Then,

$$\Delta(g) = \pi_2(h_2) \circ \Delta(x_i) \circ \pi_1(h_1) = 0$$

and so we have that $\Delta = 0$ identically. For surjectivity, assume that $\{\phi_i\}_{i=1}^r$ is a collection of intertwiners $\phi_i : \pi_{1,i} \to \pi_{2,i}$. Define:

$$\Delta(h_2 x_i h_1) = \pi_2(h_2) \circ \phi_i \circ \pi_1(h_1)$$

for $h_1 \in H_1$ and $h_2 \in H_2$. First, we show this is well-defined. Suppose $h_2 x_i h_1 = h_2' x_j h_1'$. Then these lie in the same (H_1, H_2) double coset, so $i = j$ by assumption. Then, $s := (h_2')^{-1} h_2 = x_i h_1' h_1^{-1} x_i^{-1} \in H_2 \cap x_i H_1 x_i^{-1} = S_i$. So,

$$\begin{aligned}
\pi_2(h_2) \circ \phi_i \circ \pi_1(h_1) &= \pi_2(h_2') \circ \pi_2((h_2')^{-1}) \circ \pi_2(h_2) \circ \phi_i \circ \pi_1(h_1) \\
&= \pi_2(h_2') \circ \pi_2(s) \circ \phi_i \circ \pi_1(h_1) \\
&= \pi_2(h_2') \circ \pi_{2,i}(s) \circ \phi_i \circ \pi_1(h_1) \\
&= \pi_2(h_2') \circ \phi_i \circ \pi_{1,i}(s) \circ \pi_1(h_1) \\
&= \pi_2(h_2') \circ \phi_i \circ \pi_1(x_i^{-1} s x_i h_1) \\
&= \pi_2(h_2') \circ \phi_i \circ \pi_1(h_1')
\end{aligned}$$

as desired. Second, it is clear that Δ equation (1.4), and so $\Delta \in \mathrm{Hom}_G(V_1^G, V_2^G)$ with $\Phi(\Delta) = (\phi_1, \ldots, \phi_r)$. This shows surjectivity and completes the proof. □

Exercise (4.1.3).

Proof. Let $A \in GL_n(F)$, and consider $tI - A$, a matrix with entries in the PID $F[t]$. We know that this matrix has a unique Smith normal form S, and that it is obtained via row and column operations. That is, there are matrices P and Q in $GL_n(F[t])$ with $P(tI - A)Q = S$.

Now, $tI - A^T$ also has a unique Smith normal form. But note:

$$Q^T(tI - A^T)P^T = Q^T(tI - A)^T P^T = (P(tI - A)Q)^T = S^T = S$$

since tI and S are diagonal. Hence, the Smith normal form for $tI - A^T$ is also S, since Q^T and P^T are invertible.

So, we conclude that $tI - A$ and $tI - B$ are equivalent matrices, whence A and B are similar. □

Exercise (4.1.4).

Proof. This is very direct:

$$\begin{aligned}
\langle \overline{x_1}, \overline{y} \rangle \langle \overline{x_2}, \overline{y} \rangle &= \chi(x_1 y x_1^{-1} y^{-1}) \chi(x_2 y x_2^{-1} y^{-1}) \\
&= \chi((x_1 y x_1^{-1} y^{-1})(x_2 y x_2^{-1} y^{-1})) \\
&= \chi(x_1 (x_2 y x_2^{-1} y^{-1}) y x_1^{-1} y^{-1}) \\
&= \chi(x_1 x_2 y x_2^{-1} x_1^{-1} y^{-1}) \\
&= \langle \overline{x_1 x_2}, \overline{y} \rangle
\end{aligned}$$

where we've used that the right commutator is in the center. The second follows from the first and the fourth, as:

$$\langle \overline{x}, \overline{y_1 y_2} \rangle = \langle \overline{y_1 y_2}, \overline{x} \rangle^{-1} = (\langle \overline{y_1}, \overline{x} \rangle \langle \overline{y_2}, \overline{x} \rangle)^{-1} = \langle \overline{x}, \overline{y_1} \rangle \langle \overline{x}, \overline{y_2} \rangle$$

as desired. For the third,

$$\langle \overline{x}, \overline{x} \rangle = \chi(xxx^{-1}x^{-1}) = \chi(1) = 1$$

and for the fourth,

$$\langle \overline{y}, \overline{x} \rangle^{-1} = \chi(yxy^{-1}x^{-1})^{-1} = \chi(xyx^{-1}y^{-1}) = \langle \overline{x}, \overline{y} \rangle$$

completing the computation. □

Exercise (4.1.5).

Proof. I drop the bars for convenience. Let A be a maximal isotropic subgroup of H/Z. To show that it is polarizing, suppose that $x \in H/Z$ is such that $\langle x, y \rangle = 1$ for all $y \in A$. We seek to show $x \in A$. Consider the subgroup $B = A \langle x \rangle \subseteq H/Z$. Since $A \subseteq B$, it suffices to show that B is isotropic, for then $B = A$ by maximality of A.

Note that any element of B is of the form ax^n for $a \in A$ and $n \in \mathbb{Z}$. Then,

$$\langle ax^n, bx^m \rangle = \langle a, b \rangle \langle a, x^m \rangle \langle x^n, b \rangle \langle x^n, x^m \rangle = \langle a, x \rangle^m \langle x, b \rangle^n = (1^{-1})^m (1)^n = 1$$

and so B is indeed isotropic. □

Exercise (4.1.6).

Proof. As suggested in the hint, let Z_0 be the kernel of χ_0. Then, if $x, y \in A$, we have

$$\chi_0(xyx^{-1}y^{-1}) = \langle \overline{x}, \overline{y} \rangle = 1$$

since A is isotropic. So, $xyx^{-1}y^{-1} \in Z_0$, so xZ_0 and yZ_0 commute in A/Z_0. These are arbitrary, and so A/Z_0 is abelian.

But then, χ_0 factors through $Z \to Z/Z_0$, and so χ_0 is a character of Z/Z_0, whence it can be extended to the finite abelian overgroup A/Z_0. Abusing notation and referring to this map as χ_0 as well, we finally get an extension to A by precomposing with the quotient map $A \to A/Z_0$ as desired. □

Exercise (4.1.7).

Proof. We've already noted that A, B are normal subgroups since $\overline{A}, \overline{B}$ are normal subgroups of $\overline{H}$, since it is abelian. Then clearly AB is normal as well.

Now, define $\chi : A \cap B \to \mathbb{C}$ by $\chi(s) = \chi_B(s)\chi_A(s^{-1})$. This is a character, since

$$\chi(st) = \chi_B(st)\chi_A((st)^{-1}) = \chi_B(s)\chi_B(t)\chi_A(t^{-1})\chi_A(s^{-1}) = \chi(s)\chi(t)$$

We use this to show uniqueness. Suppose x is such that $\chi_B(s) = \chi_A(x^{-1}sx)$ for all $s \in A \cap B$. Then,

$$\chi(s) = \chi_B(s)\chi_A(s^{-1}) = \chi_A(x^{-1}sx)\chi_A(s^{-1}) = \chi_A(x^{-1}sxs^{-1}) = \chi_0(x^{-1}sxs^{-1}) = \langle x^{-1}, s \rangle$$

So, x^{-1} is determined modulo $(A \cap B)^{\perp} = A^{\perp}B^{\perp} = AB$ since A, B are polarizing.

Finally, we show existence. FINISH

□

Exercise (4.1.8).

Proof. □

Exercise (4.1.9).

Proof. Suppose we have a 2-cocycle $\sigma : G \times G \to A$ and a 1-chain $\phi : G \to A$ with coboundary $\delta\phi$. We wish to show $\tilde{G}_\sigma \cong \tilde{G}_{\sigma \cdot \delta\phi}$. Define the map $f : \tilde{G}_\sigma \to \tilde{G}_{\sigma\cdot\delta\phi}$ by:

$$f(g, a) = (g, a\phi(g)^{-1}\phi(1))$$

This is a group homomorphism since

$$\begin{aligned} f((g,a)(g',a')) &= f(gg', aa'\sigma(g,g')\sigma(1,1)^{-1}) \\ &= (gg', aa'\sigma(g,g')\sigma(1,1)^{-1}\phi(gg')^{-1}\phi(1)) \\ &= (g, a\phi(g)^{-1}\phi(1))(g', a'\phi(g')^{-1}\phi(1)) \\ &= f(g,a)f(g',a') \end{aligned}$$

If (g, a) is in the kernel of f, then since the first coordinate is preserved, we get $g = 1$, and then the second coordinate gives $1 = a\phi(1)^{-1}\phi(1) = a$, so $a = 1$ as well. I.e. f injects. On the other hand, it also clearly surjects. Indeed, for $g \in G$ and $a \in A$, we have:

$$f(g, a\phi(g)\phi(1)^{-1}) = (g, a\phi(g)\phi(1)^{-1}\phi(g)^{-1}\phi(1)) = (g, a)$$

So, we have furnished an isomorphism $\tilde{G}_\sigma \to \tilde{G}_{\sigma\cdot\delta\phi}$ as desired.

Now, suppose $1 \to A \xrightarrow{i} \tilde{G} \xrightarrow{q} G \to 1$ is a central extension of G by A. Since q surjects, we can choose a function (not a group homomorphism) $s : G \to \tilde{G}$ with $q(s(g)) = g$; for convenience, choose $s(1) = 1$. Now, for $g, g' \in G$, note that

$$q(s(gg')^{-1}s(g)s(g')) = q(s(gg'))^{-1}q(s(g))q(s(g')) = (gg')^{-1}gg' = 1$$

So, $s(gg')^{-1}s(g)s(g')$ is in the kernel of q, which equals the image of i. So, let $\sigma(g, g')$ be any element of A satisfying $i(\sigma(g, g')) = s(gg')^{-1}s(g)s(g')$; again, for convenience, since $i(\sigma(1, 1)) = s(1)^{-1}s(1)s(1) = s(1) = 1$, we may choose $\sigma(1, 1) = 1$. For this map $\sigma : G \times G \to A$, we construct a group homomorphism $f : \tilde{G}_\sigma \to \tilde{G}$ by:

$$f(g, a) = s(g)i(a)$$

This is indeed a group homomorphism since:

$$\begin{aligned} f((g,a)(g',a')) &= f(gg', aa'\sigma(g,g')\sigma(1,1)^{-1}) \\ &= s(gg')i(aa'\sigma(g,g')) \\ &= s(gg')i(\sigma(g,g'))i(a)i(a') \\ &= s(g)i(a)s(g')i(a') \\ &= f(g,a)f(g',a') \end{aligned}$$

where we have used liberally the fact that the image of i lies in the center of $\tilde{G}$. This gives the diagram:

$$\begin{array}{ccccccccc} 1 & \longrightarrow & A & \longrightarrow & \tilde{G}_\sigma & \longrightarrow & G & \longrightarrow & 1 \\ & & \downarrow{\scriptstyle \mathrm{id}_A} & & \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle \mathrm{id}_G} & & \\ 1 & \longrightarrow & A & \xrightarrow{i} & \tilde{G} & \xrightarrow{q} & G & \longrightarrow & 1 \end{array}$$

We wish to show that this diagram commutes and that f is an isomorphism. For $a \in A$, we have $f(1,a) = s(1)i(a) = i(a) = i(\mathrm{id}_A(a))$, giving commutativity of the first square. For $(g,a) \in \tilde{G}_\sigma$, we have $q(f(g,a)) = q(s(g)i(a)) = q(s(g))q(i(a)) = g \cdot 1 = g$, giving commutativity of the second square. But now f is an isomorphism by the five lemma.

Finally, we wish to show uniqueness: if $\tilde{G}_\sigma$ is equivalent to $\tilde{G}_\tau$, then $\sigma = \tau \cdot \delta\phi$ for some 1-chain ϕ. Suppose that $f : \tilde{G}_\sigma \to \tilde{G}_\tau$ is an isomorphism fitting into a commutative diagram as above. Then $f(g,a) = (g, \pi(g,a))$ for some function $\pi : G \times A \to A$ by commutativity of the second square, and $\pi(1,a) = a$ by commutativity of the first. Then, for $g, g' \in G$:

$$\begin{aligned}(gg', \pi(gg',1)\sigma(g,g')\sigma(1,1)^{-1}) &= (gg', \pi(gg',1))(1, \sigma(g,g')\sigma(1,1)^{-1}) \\ &= f(gg',1)f(1,\sigma(g,g')\sigma(1,1)^{-1}) \\ &= f((gg',1)(1,\sigma(g,g')\sigma(1,1)^{-1})) \\ &= f(gg', \sigma(g,g')\sigma(1,1)^{-1}) \\ &= f((g,1)(g',1)) \\ &= f(g,1)f(g',1) \\ &= (g,\pi(g,1))(g',\pi(g',1)) \\ &= (gg', \pi(g,1)\pi(g',1)\tau(g,g')\tau(1,1)^{-1})\end{aligned}$$

So, for $\phi(g) = \pi(g,1)\sigma(1,1)\tau(1,1)^{-1}$:

$$\sigma(g,g') = \tau(g,g')\pi(g,1)\pi(g',1)\pi(gg',1)^{-1}\sigma(1,1)\tau(1,1)^{-1} = \tau(g,g')\phi(g)\phi(g')\phi(gg')^{-1} = \tau(g,g')\cdot\delta\phi(g,g') = (\tau\cdot\delta\phi)(g,g')$$

completing the proof. □

Exercise (4.1.10).

Proof. We've chosen $\omega(g)$ to be an intertwiner between the H-reps (π, W) and (π_g, W). So, for $h \in H$, we have $\omega(g)\pi(h) = \pi_g(h)\omega(g) = \pi({}^g h)\omega(g)$ i.e. $\pi({}^g h) = \omega(g)\pi(h)\omega(g)^{-1}$.

From this, we can see that ω is a homomorphism, for if $g, g' \in G$, then

$$\omega(g)\omega(g')\pi(h)\omega(g')^{-1}\omega(g)^{-1} = \omega(g)\pi({}^{g'}h)\omega(g)^{-1} = \pi({}^g({}^{g'}h)) = \pi({}^{gg'}h) = \omega(gg')\pi(h)\omega(gg')^{-1}$$

So, $\omega(gg')^{-1}\omega(g)\omega(g')$ is an intertwiner from π to itself, and by Schur, this means that $\omega(g)\omega(g')$ and $\omega(gg')$ differ by a constant multiple. In other words, $\omega(g)\omega(g') = \omega(gg')$ in $PGL(W)$. □

Exercise (4.1.11).

Proof. Let $\alpha = (x,y,z) \in H$ be in the kernel of the pairing induced by χ_0. To show that χ_0 is generic, we wish to show that $\alpha \in Z$, i.e. $x = y = 0$. Note that $\alpha^{-1} = (-x,-y,-z)$. Consider first $\beta = (0, ax, 0)$ for $a \in F$. Then, $\beta^{-1} = (0,-ax,0)$ and

$$1 = \langle \alpha, \beta \rangle = \chi_0(\alpha\beta\alpha^{-1}\beta^{-1}) = \chi_0((x, ax+y, z+B(x,ax))(-x,-ax-y,-z+B(-x,-ax))) = \psi(2aB(x,x))$$

If $x \neq 0$, then $B(x,x)$ is nonzero by nondegeneracy of the form, but then $2aB(x,x)$ varies over all elements of F as a varies. This would then imply that ψ is the trivial character, which we have assumed not to be true. Hence $x = 0$, i.e. $\alpha = (0,y,z)$. Similarly, now consider $\beta = (ay,0,0)$ so $\beta^{-1} = (-ay,0,0)$ and compute:

$$1 = \langle \alpha, \beta \rangle = \chi_0((ay, y, z - B(ay,y))(-ay,-y,-z-B(-ay,-y))) = \psi(-2aB(y,y))$$

which implies that $y = 0$ by the same argument. So, $\alpha \in Z$ as desired.

Now, let's show that A is polarizing. First, note that

$$(x,0,z),(x',0,z') = (x+x',0,z+z') = (x',z')(x,z)$$

so that A is abelian. But then half of the claim is obvious: for $\alpha, \beta \in A$, we have $\langle \alpha, \beta \rangle = \chi_0(\alpha\beta\alpha^{-1}\beta^{-1}) = \chi_0(0,0,0) = 1$. Conversely, suppose that $\alpha = (x,y,z) \in A^\perp$. We wish to show that $y = 0$. Note that for $\beta = (ay,0,0)$,

$$1 = \langle \alpha, \beta \rangle = \psi(-2aB(y,y))$$

so that $2aB(y,y) = 0$ for all $a \in F$, and as before, this gives $B(y,y) = 0$ and so $y = 0$ as desired. □

Exercise (4.1.12).

Proof. □

Exercise (4.1.13).

Proof. □

Exercise (4.1.14).

Proof. □

Exercise (4.1.15).

Proof. □

Exercise (4.1.16).

Proof. □

Exercise (4.1.17).

Proof. □

Exercise (4.1.18).

Proof. □

Exercise (4.1.19).

Proof. □

Exercise (4.1.20).

Proof. □