Numerical exploration
in sphere packing,
Fourier analysis, and physics

Exercises 4

Henry Cohn

PCMI 2022

① Using a computer algebra system, play w/ modular forms to get comfortable w/ them.

Can you plot them? (See Lowry-Duda's blog.)

Where do they vanish?

What do their q-series look like?

No specific goals, just to get familiar w/ modular forms as concrete objects you can manipulate.

② Fill in the missing explicit calculations in the construction of g in the notes.

(See the Fields medal laudatio text for Viazovska if you'd like a few more details.)

③ What happens if you try this for other functions in the interpolation basis?

Don't worry if you get stuck. The goal is just to see what happens, what you can adapt and what needs further ideas.

④ For which N does there exist a smooth function $p: (0, \infty) \to \mathbb{R}$ such that $(-1)^k p^{(k)} \geq 0$ for $0 \leq k \leq N$ but E_8 is not a ground state for the potential function p?

(For example, the beginning of the notes gets $N = 2$.)

I don't know the answer or how hard it might be, but I'd love to hear what you come up with.